



**INFLUENCE OF EXCITABILITY ON SCROLL WAVES IN A SIMULATED
EXCITABLE MEDIA**

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Abstract

In three-dimensional reaction-diffusion systems, a scroll wave rotates around one-dimensional phase singularity center, called the filament. Normally, scroll waves can collapse and disappear when they hit the boundary of the medium. In this work we consider the wave period, wavelength and wave velocity of scroll waves with different excitabilities of the medium in the Oregonator model. When the excitability grows, both wave period and wavelength decline. However, the wave velocity increases with the excitability. In addition, we reconstruct the scroll wave in three-dimension to observe their structures.

Keywords: Scroll wave, Excitability, the Oregonator model

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1. Introduction

Scroll waves occurring in three-dimensional (3D) excitable media have been discovered in many biological and chemical systems including CO oxidation on a platinum surface [1], cell aggregation in slime mold colonies [2], concentration waves in Belousov-Zhabotinsky (BZ) reaction [4, 5]. The presence of scroll waves in cardiac tissue causes cardiac arrhythmias, such as tachycardia and fibrillation [6]. A scroll wave rotating around the organizing center is called the filament which is the line connecting all a singular point along the scroll wave. Naturally, the free scroll waves drift and collapse when they hit the boundary of the system. However, this phenomenon is suppressed if they pin to unexcitable heterogeneities, such as veins or scars [3, 7].

Excitability of the media have been widely investigated in different systems. For the Ferroin-catalyzed BZ reaction with pyrogallol [PG], the excitability increases with both $[H_2SO_4]$ and $[NaBrO_3]$ but decreases with [PG] [8]. Moreover, period and wavelength of spiral waves increase with increasing [PG] or lowering $[H_2SO_4]$. All recipes support meandering of spiral tips [9]. For the classical BZ reaction with malonic acid, excitabilities are varied from low to high ($[H_2SO_4] = 80-280$ mM), both spiral core and wave period simultaneously decrease. In addition, the simulations confirm the experiments, when the parameter $1/\varepsilon$ is increased ($1/\varepsilon=20-200$) [10]. Furthermore, the simple scroll waves lead to

semiturbulence and full turbulence (corresponding to $[H_2SO_4] = 0.5$ M and 0.6 M, respectively) [11]. Corresponding to simulations, the parameter $1/\varepsilon$ is changed.

In this article, the influence of excitabilities of system on scroll waves is investigated in the Oregonator model. The parameter $1/\varepsilon$ indicated the excitability of the medium is varied. The properties of waves (e.g. wave period, wavelength, and wave speed) are considered.

2. Materials and Experiment

The two-variable Oregonator model [12] is performed to describe the three-dimensional excitable media. The dynamic of the model depends on two variables u and v (corresponding to the concentration of $HBrO_2$ and the catalyst, respectively).

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{1}{\varepsilon} \left(u - u^2 - fv \frac{u-q}{u+q} \right) + D_u \nabla^2 u, \\ \frac{\partial v}{\partial t} &= u - v + D_v \nabla^2 v \end{aligned} \quad (1)$$

The parameters of rigidly rotating scroll waves are chosen as in Ref. [13]: $q = 0.002$, $f = 1.4$, diffusion coefficients $D_u = 1.0$ and $D_v = 0.6$. The ionic mobilities M_u and M_v are chosen -0.1 and 2.0, respectively. The $1/\varepsilon$ (parameter of the excitability) is varied by 60-180. The simulations are performed using Euler method with a 27-point of the three-dimensional Laplacian operator. The

uniform grid space is set $\Delta x = \Delta y = \Delta z = 0.0125$ system unit (s.u.) and a time step $\Delta t = 4.7 \times 10^{-5}$ time unit (t.u.) as required for numerical stability [$\Delta t \leq (3/8)(\Delta x)^2$ [14]. For the size of system is $5 \times 5 \times 5$ s.u. (corresponding to $400 \times 400 \times 400$ grid points) and the system is no-flux conditions.

To initiate a scroll wave, a planar wave is triggered in the middle of the medium with size of $5 \times 5 \times 5$ s.u. Then a half of it is set to unexcitable medium and the planar wave has two ends. One wave end propagates and hit the boundary, the other one end reaches the center of the system. The wave curls and rotates in the middle of the medium. A scroll wave is completely formed.

3. Results and Discussion

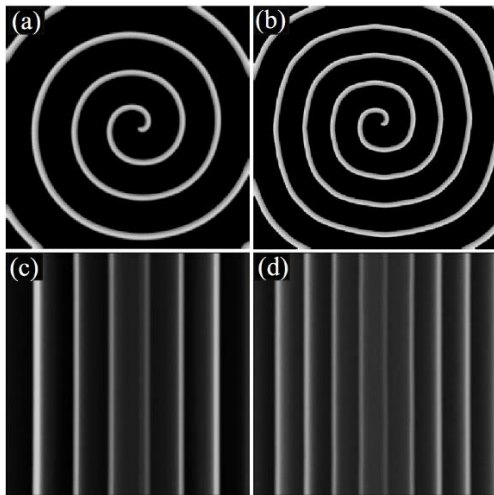


Figure 1 Free scroll waves in the Oregonator model with different of excitability. (a) and (b) are lateral side of scrolls ($1/\varepsilon$ are 60 and 180). (c) and (d) are top side ($1/\varepsilon$ are 60 and 180) at $t = 0.047$ t.u.

Fig.1 illustrates examples of free scroll waves with different wavelengths observed in our

simulations both lateral and top views. A free scroll wave with low excitability ($1/\varepsilon = 60$) in Fig. 1(a) and 1c has a wavelength $\lambda = 0.612$ s.u.. For higher excitability ($1/\varepsilon = 180$) as shown in Fig 1(a) and 1(d), has a wavelength $\lambda = 0.462$ s.u.. As the results show that wavelength of the low excitability $1/\varepsilon$ is greater than that of high excitability $1/\varepsilon$.

The examples of structure of scroll waves are shown in Fig. 2. Wavelength (blue color) of both Fig. 2(a) and 2(b) is straight and uniform. The wavelength of Fig. 2 (a) is larger than Fig 2(b). Because the excitability of the medium of Fig. 2(a) is lower than Fig. 2(b)

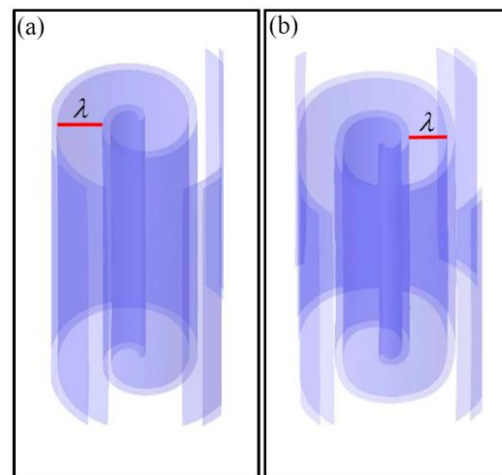


Figure 2 Example of the structure 3D scroll waves. with (a) $1/\varepsilon = 60$ and (b) $1/\varepsilon = 180$, respectively. Wavelength in (a) has 0.00865 s.u. and wavelength in (b) has 0.00587 s.u.

The dynamic behavior of the medium far from the spiral tip is presented in Fig. 3. The results show that both wave period T and wavelength λ simultaneously decrease, while the excitability of the medium $1/\varepsilon$ increase. The decay rate of wave

period T and wavelength λ are -0.0231 t.u. and -1.21×10^{-3} s.u., respectively. In contrast, the $1/\varepsilon$ effects the wave velocity increases. The growth rate is 0.139 s.u. t.u. $^{-1}$.

The results of properties of waves agree with the influence of wave in the BZ reaction with pyrogallol recipe [9]. The results demonstrated that both wave period and wavelength of the fronts grow. However, the wave speed increase with $1/\varepsilon$

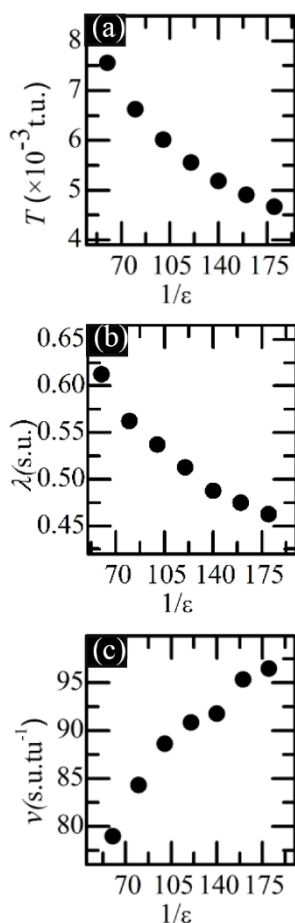


Figure 3 Properties of scroll waves versus excitability of the medium. (a) wave period (b) wavelength and (c) wave velocity.

4. Conclusions

We have presented an investigation of influence of excitabilities on the properties of scroll wave in the Oregonator model. The results demonstrated that both wave period and wavelength concurrent decline, when the excitability raises. However, the wave velocity increases with the excitability. As this results, we may further study the dynamics of scroll waves pinned to obstacles with different excitability of the medium.

5. References

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