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BEHAVIOR OF PINNED SCROLL WAVES WITH DIFFERENT EXCITABILITY IN A SIMULATED EXCITABLE MEDIA

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Abstract

The emerging of pinning phenomena is described in many dynamical systems such as superconductivity and super fluidity. The dynamics of pinned scroll waves play an important role in the human health causing longer tachycardia. In this research, we present an investigation the properties parameter (i.e. wave period, wavelength and wave velocity) of pinned scroll waves with different excitability of the media in the two-variable Oregnator model. The cylindrical obstacle is fixed constant. We found both wave period T and wavelength λ decrease with increasing the excitability. However, the wave velocity increases with excitability.

Keywords: Scroll wave, Excitability, the Oregonator model

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1. Introduction

The presence of nonlinear spatiotemporal patterns in excitable media have been discovered in chemical and biological such as CO oxidation on a platinum surface [1], cell aggregation in slime mold colonies [2], concentration waves in Belousov-Zhabotinsky (BZ) reaction [3, 4]. The existence of the patterns in cardiac tissue leads to cardiac arrhythmias by considering hart as two-dimensional (2D) medium [5, 6]. In case of three-dimensional (3D) medium or scroll waves, the ventricular fibrillation can occur. Generally, free scroll waves can be self-annihilation in short time. However, if they anchor with localized heterogeneities such as veins and scars in heart, their lives are longer.

A scroll wave can be described as a stack of rotating spiral waves that the spiral cores are organized by line connecting. This line is defined as filament, a line that the scroll wave rotates around [7-9]. A positive tension is stable for a straight filament, but the negative tension leads wave to turbulence [9, 10]. However, the turbulence waves are suppressed if they pin with the heterogeneities [11].

The properties of spiral waves pinned to defect were studied as model of unexcitable disks which investigated the effects of obstacle size. Period and velocity of pinned spiral waves increase when the hole is enlarged [12]. The formula in Ref. [13] demonstrated that velocity of waves at the periphery of obstacle increased with obstacle diameter. Both unexcitable and partially excitable circles in simulations caused the period of spiral wave to rise [14]. For circles and rectangles, the propagating parameters (i.e., wavelength, wave period and wave velocity) increased with the circumference of obstacles [15]. For scroll waves in simulations, period linearly increased with the obstacle radii [11].

In this article, the influence of excitabilities of system on pinned scroll waves is investigated in the Oregonator model. The parameter ε^{-1} indicated the excitability of the medium is varied. The cylindrical obstacle with fixed both diameter and length is placed to observe the properties of pinned waves at the middle of system.

2. Materials and Experiment

Numerical simulations are performed using the two-variable Oregonator model, as in Eq. (1) [16], one equation describing the dynamics of activator *u* and the other inhibitor *v*:

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon}\left(u - u^2 - fv\frac{u-q}{u+q}\right) + D_u \nabla^2 u, \qquad \frac{\partial v}{\partial t} = u - v + D_v \nabla^2 v \tag{1}$$

where the parameters are selected as in Ref. [16]: q = 0.002, *f* = 1.4, diffusion coefficients D_u = 1.0 and D_v = 0.6. We vary parameter of excitability $1/\varepsilon$ from 60 to 180. These parameters explain the rigidly rotating scroll waves.

An explicit Euler method with a 27-point of the three-dimensional Laplacian operator is used to calculated the variables *u* and *v* in Eq. (1). This method operates on a discrete system of a dimensionless size = $5\times5\times5$ s.u. with a uniform grid space of $\Delta x = \Delta y = \Delta z = 0.0125$ and a time step $\Delta t = 4.7\times10^{-5}$ time unit (t.u.) as required for numerical stability [$\Delta t \leq (3/8)(\Delta x)^2$ [17]. A completely unexcitable cylindrical obstacle with diameter of 0.2 s.u. is put at the middle of the system. Therefore, both the boundaries the obstacle have no-flux conditions. The creation of pinned scroll waves was described in method B in Ref. [18].

3. Results and Discussion

Fig. 1 demonstrates latter view of pinned scroll waves with different wavelengths observed in our simulations. A pinned scroll wave with low excitability ($1/\varepsilon$ = 60) in Fig. 1(a) has a wavelength λ = 0.812 s.u.. For higher excitability ($1/\varepsilon$ = 180) as shown in Fig. 1(b), a pinned scroll wave has a wavelength λ = 0.762 s.u.. As the results show that wavelength of pinned scroll waves in low excitability is greater than that of high excitability.

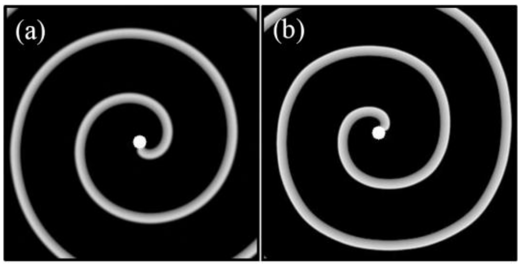

Figure 1 Scroll waves pinned to cylindrical obstacles in the Oregonator model with different excitability. (a) and (b) are latter side of pinned scrolls ($1/\varepsilon$ are 60 and 180, respectively).

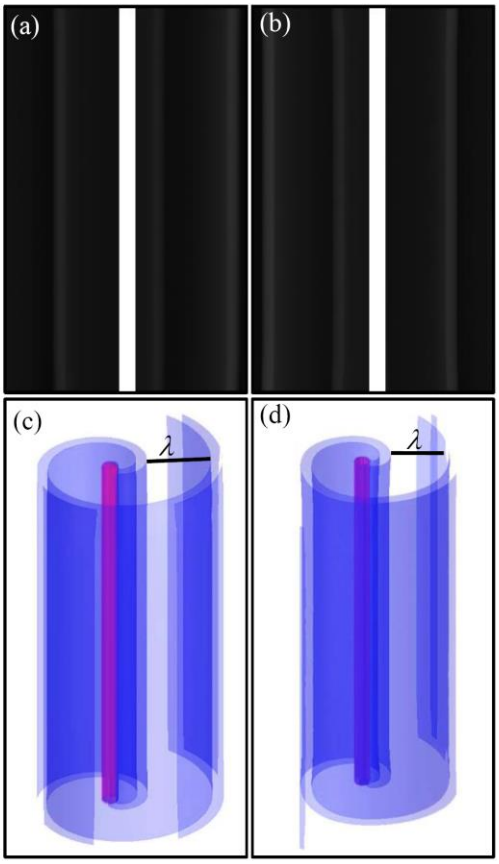

Figure 2 Latter sides of pinned scroll waves with (a) $1/\varepsilon$ = 60 and (b) $1/\varepsilon$ =180, respectively. In addition, the structures of scroll waves are shown in (c) $1/\varepsilon$ = 60 and (d) $1/\varepsilon$ =180, respectively.

Latter views of pinned scroll waves with different excitability are considered as shown in Fig. 2(a) and 2(b). Wave fronts of both excitabilities are straight and uniform. Although the obstacle is the same size, the wavelength of Fig. 2(a) is larger than Fig. 2(b). Because the excitability of the medium of Fig. 2(a) is lower than Fig. 2(b). To understanding the structure, the 3D pinned scroll waves are reconstructed to observe their structures [see Fig. 2(c) and. 2(d)]. The cylindrical obstacles (red color) are located at the middle. Scroll waves rotate and generate wave fronts (blue color) around the obstacles. The wavelength of Fig. 2(c) is greater than Fig. 2(d).

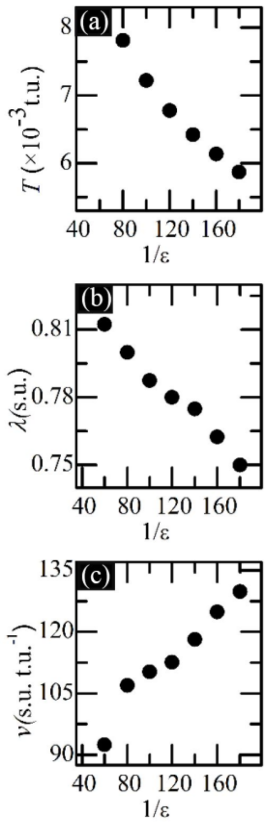

Figure 3 Properties of pinned scroll waves as a function of excitability in the Oregonator model. (a) wave period (b) wavelength and (c) wave velocity.

The properties of pinned scroll wave in our simulations are presented in Fig. 3. The scroll waves propagate around the obstacles with wavelength λ [Fig. 3(a)] and wave period [Fig. 3(b)] declining with increasing the excitability $1/\varepsilon$. The decline rate of wave period and wavelength with respect to the excitability are evaluated by linear fits as T = -2.20×10^{-4} t.u. and λ = -4.91×10^{-4} s.u., respectively. To investigate the effect of the obstacle on velocity of the scroll waves, we measure the average velocity of the wave end to another wave end far from the obstacle. The results show that the velocity increases with the $1/\varepsilon$ and the growth rate is 0.278 s.u. t.u.$^{-1}$.

4. Conclusions

An investigation the properties of scroll waves pinned to cylindrical obstacles is presented in our simulations based on the Oregonator model. The obstacle size is given but the excitability of the medium is varied from 60 - 180. The results in Figs. 1, 2 and 3 show the common features of the influence of these excitabilities on the pinned scroll waves: both wave period and wavelength decrease, when the excitability of the medium increases. In the other hand, the wave velocity increases with excitability.

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7. References

[1] Nettesheim, S., von Oertzen, A., Rotermund, H. H. and Ertl G. Reaction diffusion patterns in the catalytic CO-oxidation on Pt (110): Front propagation and spiral waves. J. Chem. Phys. 1993. 98: 9977.

[2] Siegert, F. and Weijer, C.J. Control of Cell Movement during Multicellular Morphogenesis, J. Cell Sci. 1989. 93: 325.

[3] Winfree, A. T. Spiral waves of chemical activity. Science. 1972. 175: 634.

[4] Ross, J., Müller, S. C. and Vidal, C. Chemical waves. Science. 1988. 240: 460.

[5] Jalife, J., Gray, R.A., Morley, G.E. and Davidenko, J.M. Self-organization and the dynamical nature of ventricular fibrillation, Chaos. 1998. 8:79-93.

[6] Ten Tusscher, K.H.W.J., Hren, R., Panfilov, A.V. Organization of ventricular fibrillation in the human heart, Circulation Research. 2007. 100: e87-e101.

[7] Winfree, A.T. Scroll-shaped waves of chemical activity in three dimensions, Science. 1973.181: 937-939.

[8] Luengviriya, C., Storb, U., Hauser, M.J.B. and Müller, S.C. An elegant method to study an isolated spiral wave in a thin layer of a batch Belousov-Zhabotinsky reaction under oxygen-free conditions, Phys Chem Chem Phys. 2006. 8: 1425-1429.

[9] Luengviriya, C., Storb, U., Lindner, G., Müller, S.C., Bär, M. and Hauser M.J.B. Scroll wave instabilities in an excitable chemical medium, Phys. Rev. Lett. 2008. 100: 148302.

[10] Bánsági, T., and Steinbock, O. Negative filament tension of scroll rings in an excitable system, Phy Rev E. 2007. 76: 045202(R).

[11] Spreckelsen, F., Hornung, D., Steinbock, O., Parlitz, U., Luther S. Stabilization of three-dimensional scroll waves and suppression of spatiotemporal chaos by heterogeneities, Phys Rev E. 2015. 92:042920.

[12] Tyson, J.J. and Keener, J.P. Singular perturbation theory of traveling waves in excitable media (a review), Physica D. 1988. 32: 327-361.

[13] Tanaka, M., Isomura, A., Hörning, M., Kitahata, H., Agladze, K. and Yoshikawa, K. Unpinning of a spiral wave anchored around a circular obstacle by an external wave train: Common aspects of a chemical reaction and cardiomyocyte tissue, Chaos. 2009. 19: 043114.

[14] Fu, Y.-Q., Zhang, H., Cao, Z., Zheng, B. and Hu, G. Removal of a pinned spiral by generating target waves with a localized stimulus, Phys Rev E. 2005. 72: 046206.

[15] Sutthiopad, M., Luengviriya, J., Porjai, P., Phantu, M., Kanchanawarin, J., Müller, S.C. and Luengviriya, C. Propagation of spiral waves pinned to circular and rectangular obstacles, Phys Rev E. 2015. 89: 052902.

[16] Jahnke, W. and Winfree, A.T. A survey of spiral wave behavior in the Oregonator model, Int. J. of Bif. Chaos. 1991. 1:445-466.

[17] Dowle, M., Mantel, R.M., and Barkley, D. Fast simulations of waves in three-dimensional excitable media, International Journal of Bifurcation and Chaos in Applied Sciences and Engineering. 1997. 7: 2529-2545.

[18] Phantu, M., Kumchaiseemak, N., Porjai, P., Sutthiopad, M., Müller, S.C., Luengviriya, C. and Luengviriya, J. Generation of spiral waves pinned to obstacles in a simulated excitable system, Journal of Physics: Conf. Series. 2017. 901: 012021.